Thrust Effectiveness of Oxidizer Injection in a Convergent-Divergent Nozzle

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An analysis defining the thrust effectiveness obtained by oxidizer injection into fuel-rich exhaust products in the divergent section of a convergent-divergent nozzle is presented. This concept could be applied to a liquid bipropellant system that yields a stoichiometric temperature of the reaction products too high to be tolerated in the region of the nozzle throat. For such systems, the stoichiometric temperature of the reaction products could be attained in divergent section of the nozzle where the convective heating is less severe than at the throat. Based on the same maximum allowable gas temperature at the nozzle throat, the results obtained from an idealized flow model of this concept indicate a potentially greater increase in nozzle thrust by oxidizer injection in the nozzle exit cone than by the same propellant mass addition in the combustion chamber.

Nomenclature

A	=	flow area
C_p	=	specific heat at constant pressure
F/F_{REF}	=	thrust ratio, nozzle thrust at specific conditions per basic nozzle thrust
h	=	heat of reaction per pound oxidizer
$\dot{H}$	=	stagnation enthalpy flow rate
$\dot{m}$	=	mass flow rate
M	=	Mach number
(O/F)	=	oxidizer-fuel ratio
P	=	pressure
Q	=	nondimensional stagnation enthalpy flow increment, $\dot{H}_2 - \dot{H}_1/\dot{H}_1$
R	=	radius
$\mathcal{R}$	=	gas constant
T	=	temperature
U	=	velocity
W	=	nondimensional mass flow increment, $\dot{m}_2 - \dot{m}_1/\dot{m}_1$
x	=	axial distance measured from A^*
γ	=	ratio of specific heats
θ	=	nozzle half angle
ρ	=	density

Subscripts

1	=	axial location 1
2	=	axial location 2
*	=	nozzle throat
e	=	nozzle exit
0	=	stagnation condition

Introduction

HIGH impulse propellant combinations currently being studied for rocket motor applications have combustion temperatures as high as 9000°R at the optimum oxidizer to fuel mixture ratio. The flow of exhaust gases in a rocket motor at such extreme stagnation temperatures will result in severe heating rates at the nozzle throat, causing deterioration or structural failure of the nozzle unless limited to relatively short operating durations.

The operating duration of a rocket motor utilizing such a propellant could be extended by using a fuel-rich chamber mixture ratio, thereby reducing the temperature of the combustion products, as well as providing a less corrosive chemical environment in the combustion chamber and nozzle throat. Downstream of the nozzle throat, where the heating rates are reduced because of a reduced heat-transfer coefficient and a reduced gas temperature, an increase in the stagnation temperature could be tolerated. By injection of oxidizer through the nozzle wall in a continuous fashion over some appreciable nozzle length downstream of the throat, the optimum oxidizer-fuel mixture ratio could be attained within the nozzle.

Another potential application for the concept of oxidizer injection in the nozzle might be to reduce thrust losses caused by flow overexpansion in high-expansion ratio nozzles operating at low altitudes.[1] Injecting oxidizer into fuel-rich exhaust products downstream of the throat in such nozzles would result in a reduction of the flow Mach number and an increase in the static pressure. Proper control of the injection rate could provide effective altitude compensation.

It is the objective of the following analysis to determine the thrust effectiveness provided by the process of oxidizer injection into the divergent section of a convergent-divergent nozzle.

Analyses

Consider the conical convergent-divergent (C-D) nozzle shown in Fig. 1. The fuel-rich exhaust products are assumed to expand isentropically from the combustion chamber to axial location A_1. Oxidizer is injected at a uniform rate per unit surface area perpendicular to the nozzle axis between axial locations A_1 and A_2. Isentropic expansion of the exhaust products is then resumed from A_2 to the exit plane A_e. It is desired to evaluate the thrust provided by the oxidizer injection between A_1 and A_2.

The assumptions defining the flow model are: 1) the flow is steady, one-dimensional, and inviscid; 2) the exhaust gases are perfect gases with constant specific heats and

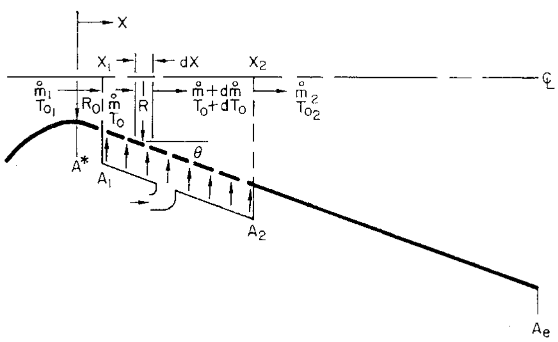

Fig. 1 Convergent-divergent (C-D) nozzle.

Received December 24, 1963; revision received May 15, 1964.

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molecular weights; 3) the flow is fully reacted at each axial location; and 4) the heat and mass are added reversibly.

This rather idealized flow model will provide results that represent the upper limit of performance that could be achieved by the oxidizer injection concept.

Considering the preceding assumptions, the equations defining the flow system are as follows:

Conservation of Mass

$$\dot{m} = \rho A u \tag{1a}$$

$$\dot{m} = \dot{m}_1 + \int_{x_1}^{x} \left(\frac{d\dot{m}}{dx}\right) dx \tag{1b}$$

Conservation of Energy

$$T_0 = T\left(1 + \frac{\gamma - 1}{2} M^2\right) \tag{2a}$$

$$\dot{m} c_p T_0 = \dot{m}_1 c_p T_{01} + \int_{x_1}^{x} \left(\frac{dH}{dx}\right) dx \tag{2b}$$

Stream Momentum

$$-A \frac{dP}{dx} = \frac{d}{dx} (\dot{m} u) \tag{3}$$

Equation of State

$$P = \rho \mathcal{R} T \tag{4}$$

Enthalpy-Mass Relation

$$\frac{d}{dx} (\dot{H}) = h \frac{d}{dx} (\dot{m}) \tag{5}$$

Geometry Relation

$$A = \pi (R_0 + x \tan\theta)^2 \tag{6}$$

By combining Eqs. (1a, 2a, 3, and 4) and their derivatives, and introducing Mach number, the variables P, T, ρ, and U may be eliminated and the following relation is obtained[2]:

$$\frac{dM^2}{M^2} = \frac{1 + [(\gamma - 1)/2]M^2}{1 - M^2} \left[(1 + \gamma M^2)\left(2 \frac{d\dot{m}}{\dot{m}} + \frac{dT_0}{T_0}\right) - 2 \frac{dA}{A}\right] \tag{7}$$

Equation (7) defines the variation of Mach number with mass addition, stagnation temperature change, and flow area change in the nozzle. The parameters $d\dot{m}/\dot{m}$ and dT_0/T_0 may be eliminated by relating them to the total mass and enthalpy increase between A_1 and A_2. From Eqs. (1b) and (6) it may be shown, assuming a uniform rate of oxidizer injection, that

$$\frac{d\dot{m}}{\dot{m}} = \left[\frac{WA/A_1}{(A_2/A_1 - 1) + W(A/A_1 - 1)}\right] \frac{dA}{A}$$

where

$$W \equiv \frac{\text{total mass flow increment between } A_1 \text{ and } A_2}{\text{mass flow in combustion chamber}}$$

Similarly, from Eqs. (2b, 5, and 6), it may be shown that

$$\frac{dT_0}{T_0} = \left[\frac{QA/A_1}{(A_2/A_1 - 1) + Q(A/A_1 - 1)} - \frac{WA/A_1}{(A_2/A_1 - 1) + W(A/A_1 - 1)}\right] \frac{dA}{A}$$

where

$$Q \equiv \frac{\text{total stagnation enthalpy flow increment between } A_1 \text{ and } A_2}{\text{enthalpy flow in combustion chamber}}$$

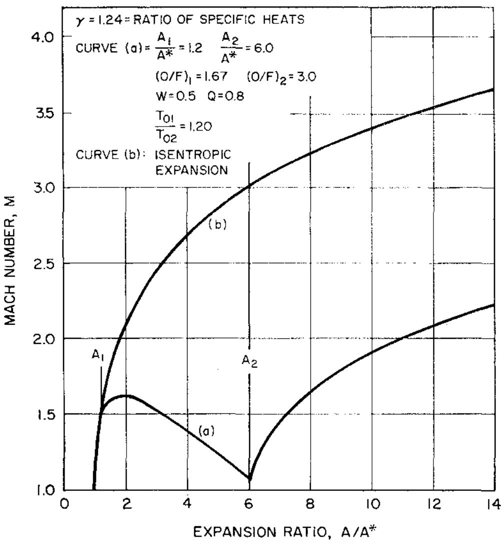

Fig. 2 Mach number variation with nozzle expansion ratio.

Substituting the preceding expression for $d\dot{m}/\dot{m}$ and dT_0/T_0 into Eq. (7) gives

$$\frac{dM^2}{M^2} = \frac{1 + [(\gamma - 1)/2]M^2}{1 - M^2} \left\{(1 + \gamma M^2) \times \left[\frac{WA/A_1}{(A^2/A_1 - 1) + W(A/A_1 - 1)} + \frac{QA/A_1}{(A_2/A_1 - 1) + Q(A/A_1 - 1)}\right] - 2\right\} \frac{dA}{A} \tag{7a}$$

It is now appropriate to relate the total mass and stagnation enthalpy increase parameters W and Q and the stagnation temperature increase T_{02}/T_{01} to the initial and final oxidizer to fuel ratios, $(O/F)_1$ and $(O/F)_2$. This may be accomplished utilizing Eqs. (1a, 2a, and 5), the definitions of W and Q, and assuming the heat of reaction per pound of oxidizer injection h to be a constant. The results obtained are

$$W = \frac{(O/F)_2 - (O/F)_1}{1 + (O/F)_1} \qquad Q = \frac{(O/F)_2 - (O/F)_1}{(O/F)_1} \tag{8}$$

$$\frac{T_{02}}{T_{01}} = \frac{1 + Q}{1 + W} = \frac{(O/F)_2[1 + (O/F)_1]}{(O/F)_1[1 + (O/F)_2]} \tag{9}$$

Equation (7a) may be solved numerically by finite difference methods to obtain the variation in Mach number between A_1 and A_2 for any preselected values of $(O/F)_1$ and $(O/F)_2$ provided the corresponding values of W and Q given by Eq. (8) are less than the critical amount to cause thermal choking of the flow. Having solved Eq. (7a) for M_2 and knowing the mass flow and stagnation temperature at A_2, all other thermodynamic variables of the flow may be computed at A_2. Then, using isentropic expansion relations, the stream momentum (vacuum thrust) at the nozzle exit plane A_e may be determined.

Discussion

The variation in Mach number with nozzle expansion ratio obtained from a numerical solution of Eq. (7a) with $W = 0.5$ and $Q = 0.8$ is shown in Fig. 2. These values of mass addition and stagnation enthalpy increase correspond to an initial oxidizer-fuel ratio of 1.67, a final oxidizer-fuel ratio of 3.0, and a 20% increase in the stagnation temperature of the combustion products between axial locations A_1 and A_2. The oxidizer was assumed to be added at a uniform rate per

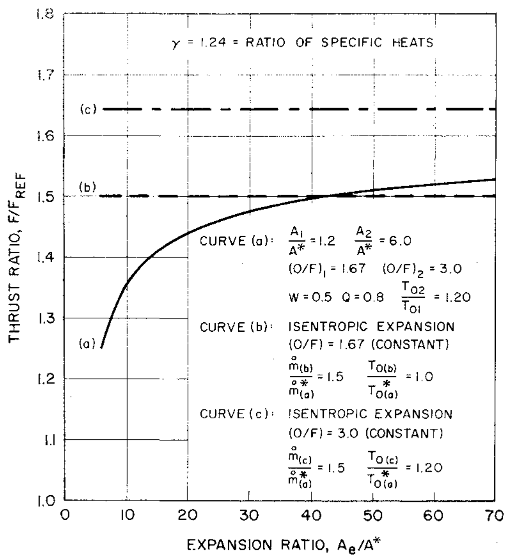

Fig. 3 Thrust ratio variation with nozzle expansion ratio.

unit surface area between an expansion ratio of $A_1/A^* = 1.2$ and $A_2/A^* = 6.0$. For comparative purposes, Fig. 2 also shows the variation in Mach number for a totally isentropic expansion process ($W = 0$ and $Q = 0$).

The effect of the process of oxidizer injection in the nozzle is to decelerate the supersonic stream by both the mechanism of mass addition and heat addition, whereas the effect of the flow area increase is to accelerate the stream. Figure 2 shows the increasing influence of the oxidizer injection in decelerating the flow as the expansion ratio increases. For the specific case shown, the Mach number is seen to reach a maximum at an expansion ratio of approximately two and then decrease toward a sonic condition up to an expansion ratio of $A_2/A^* = 6.0$. Beyond an expansion ratio of six, the flow undergoes the usual isentropic expansion process and the stream momentum, or vacuum thrust, may be computed at any axial location.

Based on the axial variation of Mach number from Fig. 2, the ratio of the thrust provided by oxidizer injection in the nozzle to the basic nozzle thrust is shown in Fig. 3 as curve (a). For comparison, the thrust ratio provided by the equivalent propellant mass addition in the combustion chamber is shown as curve (b) and by the equivalent oxidizer mass addition in the combustion chamber as curve (c). The stagnation temperature and oxidizer-fuel ratio of curve (b) are identical to those at axial location A_1 in curve (a), whereas the stagnation temperature and oxidizer-fuel ratio of curve (c) are identical to those at axial location A_2 in curve (a).

The thrust ratio provided by oxidizer injection in the divergent section of the nozzle is seen to increase with expansion ratio from 1.25 at $A_e/A^* = 6.0$ (corresponding to oxidizer injection along the entire length of the nozzle) to 1.53 at $A_e/A^* = 70$. As the expansion ratio approaches infinity, the asymptotic limit of the thrust ratio provided by the nozzle injection process approaches that of the equivalent oxidizer mass injection in the combustion chamber. As intuitively expected, the process of oxidizer injection in the nozzle is less efficient in providing thrust at all finite values of expansion ratio than the equivalent mass of oxidizer added in the combustion chamber.

If, however, the stagnation temperature of the gas at the nozzle throat is limited by nozzle heating criteria to the value corresponding to curves (a) and (b), or if because of molecular weight and dissociation effects it is not desirable to attain the stoichiometric temperature of the combustion products in the combustion chamber where the static temperature is equal to the stagnation temperature, a thrust improvement by oxidizer injection downstream of the nozzle throat, as compared to the same increase in propellant mass flow added conventionally in the combustion chamber, is indicated at the higher expansion ratios. At an expansion ratio of 60, the nozzle gross thrust with oxidizer injection in the nozzle is shown to be 1.3% higher than the thrust obtained by the same propellant mass addition in the combustion chamber.

In addition, an effect that is not reflected in Fig. 3 is that the throat area of nozzle (b) is necessarily 1.5 times greater than that of nozzle (a) in order to accommodate the additional propellant mass flow at the same chamber pressure and gas temperature. Thus, the expansion ratio of nozzle (b) is only two-thirds that of nozzle (a), assuming the same over-all length and exit areas of nozzles (a) and (b). This, of course, reduces vacuum thrust coefficient of nozzle (b) relative to that of nozzle (a). Considering this effect in the preceding numerical example results in a total thrust increase by oxidizer injection in the nozzle of 2.7% higher than the thrust obtained by the same propellant mass addition in the combustion chamber. Further improvement would have been indicated by the analysis had the oxidizer been assumed injected with a component of velocity parallel to the nozzle axis rather than normal to the nozzle axis.

It should be noted that the results of the preceding analysis are optimistic in view of the idealized flow model. This flow model provides the theoretical upper limit of thrust performance to be achieved by downstream oxidizer injected perpendicular to the nozzle axis. In reality, the relatively minor improvement in thrust performance indicated by the analysis at the higher values of nozzle expansion ratio could well be nullified by relatively slow diffusion rates, reaction rates, and irreversibilities.

References

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[2] Sharpiro, A. H., "The dynamics and thermodynamics of compressible fluid flow" (Ronald Press Co., New York, 1954), Chap. 8.